Radiative correction to the transferred polarization in elastic electron-proton scattering

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Using a method of electron structure functions, we calculated a model independent radiative correction to the recoil proton polarization for elastic electron-proton scattering. Explicit expressions for the recoil proton polarization are represented as a convolution of the electron structure functions and the hard part of the polarization-dependent contribution into the cross section. The hard part is calculated with first order radiative corrections. The obtained representation includes leading radiative corrections in all orders of perturbation theory and the main part of the second order next-to-leading radiative corrections. Numerical calculations illustrate our analytical results.

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I. INTRODUCTION

It was proposed over 25 years ago [1] that recoil proton polarization in the elastic process $\vec{e}+P\rightarrow e+\vec{P}$ can be used to measure the proton electric form factor (G_E). This method provides an alternative to the Rosenbluth separation and appears to be more sensitive to G_E in the GeV range of 4-momentum transfers (Q^2). Such measurements were done first at MIT-Bates [2] and extended later on to higher $Q^2=3.5$ GeV^2 at Jefferson Lab [3]. The latter experiment provided the first evidence of significant deviation of G_{EP} from the dipole form at higher Q^2.

In the recent Jefferson Lab experiment [3] events corresponding to the elastic process

$$\vec{e}^{-}(k_1)+P(p_1)\rightarrow e^{-}(k_2)+\vec{P}(p_2) \tag{1}$$

as well as the radiative process

$$\vec{e}^{-}(k_1)+P(p_1)\rightarrow e^{-}(k_2)+\gamma(k)+\vec{P}(p_2) \tag{2}$$

have been analyzed.

The main goal of these experiments is the measurement of the proton electric form factor G_E. It can be done because the ratio of the longitudinal polarization of the recoil proton to the transverse one in the Born approximation is proportional to the ratio G_M/G_E [1], where G_M is the well known proton magnetic form factor. Interpretation of these high-precision experiments in terms of the proton electromagnetic form factors G_M and G_E requires adequate theoretical calculations with a percent accuracy or better. Such calculations must include the first order radiative corrections (RC) to the elastic cross section (due to the radiation of real soft and virtual photons) and full analysis of the radiative events. Moreover, leading higher order corrections have to be taken into account.

The total radiative correction which has to be applied to data can be naturally divided into model-independent and model-dependent corrections. The model-independent correction can be calculated in the framework of a one-photon exchange approximation. It includes all corrections to the lepton part of interaction and insertion of vacuum polarization into the exchange photon propagator. The model-dependent correction involves additional photon-hadron coupling(s) and comes from box-type diagrams, hadronic vertex functions, etc. The current practice of data analysis in experiments on ep scattering is that the model-independent correction is taken into account with accuracy provided by theoretical calculations. Practically it means that the contribution of the radiative effects is calculated theoretically and simply subtracted from experimentally observed quantities or some Monte Carlo generators constructed on the basis of these calculations are implemented into codes of data analysis. The model-dependent correction is analyzed at the level allowed by the current knowledge of hadronic structure and its contribution is added to the systematic error due to RC. There are several reasons for that. First, the model-independent correction is the main contribution because of the smallness of electron mass. Second, the model-independent correction can be calculated without any additional assumptions. There is also a historical reason to give the most attention to the model-independent correction. A lot of measurements were done using classical formulas of Mo and Tsai, where only model-independent corrections were calculated. So for a proper comparison between the results of different experiments only the model-independent correction should be calculated and applied.

All the corresponding contributions to the model-independent correction can be unified within the framework of the electron structure function representation, which is a QED analog of the well known Drell-Yan representation [4]. This representation was applied before for the calculation of

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RC to unpolarized electron-positron annihilation [5] and deep inelastic scattering [6] cross sections. In the present work we generalize the electron structure function representation for the case of scattering of polarized particles, namely for the analysis of the recoil proton polarization in elastic ep scattering. Our analytical formulas for RC include the exact first order contribution (of the order α), the leading contribution in all orders [of the order $(\alpha L)^n$], and the main part of the second order next-to-leading contribution (of the order $\alpha^2 L$). Together with numerical integration precision, which can be optionally changed in corresponding codes, they provide the accuracy of calculated model-independent RC at the level of 0.1%.

The model-dependent correction including box-type diagrams and hadron vertex function also can give important contribution to the observables. However, it cannot be calculated using the methods developed here and will be a subject of a separate investigation. Recently, an important step in the understanding of the model-dependent correction was done in Ref. [7], where it was calculated with certain approximations. It was shown that under the made assumptions the proton vertex correction can reach 2%.[1]

This paper is organized as follows. Section II introduces definitions of electron structure functions and their contributions to the observable cross section and recoil proton polarization. Next-to-leading order (NLO) contributions to the observables are calculated in Sec. III. Section IV is devoted to the numerical analysis of radiative corrections under the kinematic conditions of the current Jefferson Lab experiments on polarization transfer in elastic ep scattering. Section V gives a summary of our results.

II. THE LEADING APPROXIMATION

The cross section of electron-proton scattering can be represented in the electron structure function (SF) method as a convolution of a two-electron SF and the hard part of the cross section that depends on the shifted 4-momenta. The electron SF account for radiation of hard collinear photons, virtual photons as well as electron-positron pairs by the initial and final electrons. This representation follows from the quasireal electron method [8] that is suitable for a description of collinear radiation.

In the problem considered here we will be interested only in the spin-dependent part of the cross section. Then the corresponding representation can be written as

$$\frac{d\sigma^{\parallel,\perp}(k_1,k_2)}{dQ^2 dy}=\int_{z_{1m}}^{1} dz_1 \int_{z_{2m}}^{1} dz_2 D^{(p)}(z_1,L)\frac{1}{z_2^2}D^{(u)}(z_2,L) \times\frac{d\sigma^{\parallel,\perp(hard)}(\hat{k}_1,\hat{k}_2)}{d\hat{Q}^2 d\hat{y}}, \quad L=\ln\frac{Q^2}{m^2}, \tag{3}$$

[1]We should note also that the noncritical application of a model-dependent correction to the data can result in a double-counting of the effect. For example, if a model for form factors is used as a result of the fit of experimental data, where a model-dependent correction was not applied for, then probably the correction should not be applied for the measurement either.

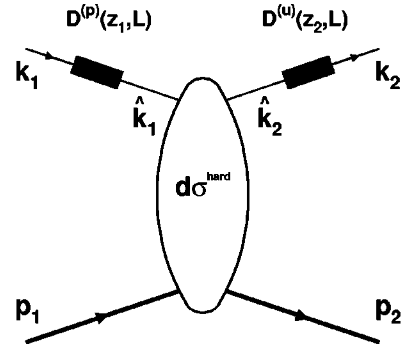

FIG. 1. Drell-Yan-like representation for the cross section of elastic ep scattering. This representation takes into account model-independent QED radiative corrections.

where m is the electron mass,

$$\hat{k}_1=z_1k_1, \quad \hat{k}_2=\frac{k_2}{z_2}, \quad Q^2=-(k_1-k_2)^2,$$

$$\hat{Q}^2=-(\hat{k}_1-\hat{k}_2)^2=\frac{z_1}{z_2}Q^2,$$

$$y=\frac{2p_1(k_1-k_2)}{V}, \quad \hat{y}=1-\frac{1-y}{z_1z_2}, \quad V=2p_1k_1. \tag{4}$$

A diagram in Fig. 1 explains the physical meaning of the Drell-Yan-like representation used to describe the QED radiative corrections in the considered process. This diagram shows that the bare electron lines have to be replaced by some effective electron lines, and the hard subprocess should be calculated for the scattering of the electron with reduced momentum $\hat{k}_1$, provided that the scattered electron has a momentum $\hat{k}_2$.

The electron structure function $D^{(p)}(z_1,L)$ is responsible for radiation by the initial polarized electron, whereas the function $D^{(u)}(z_2,L)$ describes radiation by the scattered unpolarized electron. The photon contribution into the electron structure function is the same for polarized and unpolarized cases, but the contribution due to pair production differs in the singlet channel [9]. Therefore, we can write

$$D^{(u)}(z,L)=D^{\gamma}(z,L)+D_N^{e^+e^-}+D_S^{e^+e^-(u)}, \tag{5}$$

$$D^{(p)}(z,L)=D^{\gamma}(z,L)+D_N^{e^+e^-}+D_S^{e^+e^-(p)}. \tag{6}$$

The electron structure functions $D^{\gamma}(z,L)$, $D_N^{e^+e^-}(z,L)$, and $D_S^{e^+e^-(u)}(z,L)$ satisfy Dokshitzer-Gribov-Lipatov-Altarelli-Parisi (DGLAP) equations [10]. Explicit forms of these equations were given in Ref. [5]. The iterative solution of the equation for $D^\gamma(z,L)$ defines it as the following series:

$$D^\gamma(z,L)=\delta(1-z)+\sum_{k=1}^{\infty}\frac{1}{k!}\left(\frac{\alpha L}{2\pi}\right)^k P_1(z)^{\otimes k},$$

$$\underbrace{P_1(z)\otimes\cdots\otimes P_1(z)}_{k}=P_1(z)^{\otimes k},$$

$$P_1(z)\otimes P_1(z)=\int_z^1 P_1(t)P_1\left(\frac{z}{t}\right)\frac{dt}{t},$$

$$P_1(z)=\frac{1+z^2}{1-z}\theta(1-z-\Delta)+\delta(1-z)\times\left(2\ln\Delta+\frac{3}{2}\right),\quad \Delta\ll 1, \tag{7}$$

where $P_1(z)$ is, in fact, the kernel of the DGLAP equation for $D^\gamma(z,L)$. The iterative form (7) of D^γ does not include any effects caused by pair production. The corresponding nonsinglet part of the structure function due to real and virtual pair production $D_N^{e+e-}(z,L)$ can be inserted into the iterative form of $D^\gamma(z,L)$ by replacing $\alpha L/2\pi$ on the right-hand side of Eq. (7) with the effective electromagnetic coupling

$$\frac{\alpha_{eff}}{2\pi}=-\frac{3}{2}\ln\left(1-\frac{\alpha L}{3\pi}\right), \tag{8}$$

which is an integral of the running electromagnetic constant.

The singular at $z=1$ terms in the electron structure function (7) can be summed in all orders, leading to the exponential form of both $D^\gamma(z,L)$ and $D_N^{e+e-}(z,L)$. There exist many different representations for them [10,11], but here we will use the form given in [5]

$$D^\gamma(z,L)=\frac{1}{2}\beta(1-z)^{\beta/2-1}\left[1+\frac{3}{8}\beta-\frac{\beta^2}{48}\left(\frac{1}{3}L+\pi^2-\frac{47}{8}\right)\right]-\frac{\beta}{4}(1+z)+\frac{\beta^2}{32}\left[-4(1+z)\ln(1-z)-\frac{1+3z^2}{1-z}\ln z-5-z\right],\quad \beta=\frac{2\alpha}{\pi}(L-1), \tag{9}$$

$$D_N^{e^+e^-}(z,L)=\frac{\alpha^2}{\pi^2}\left\{\frac{1}{12(1-z)}\left(1-z-\frac{2m}{\varepsilon}\right)^{\beta/2}\left(L_1-\frac{5}{3}\right)^2\times\left[1+z^2+\frac{\beta}{6}\left(L_1-\frac{5}{3}\right)\right]\right\}\theta\left(1-z-\frac{2m}{\varepsilon}\right), \tag{10}$$

where ε is the energy of the parent electron and $L_1=L+2\ln(1-z)$. The above form of the structure function $D_N^{e^+e^-}$ includes effects due to real pair production only. The correction caused by virtual pairs is included in D^γ. Note that the terms containing α^2L^3 cancel each other in the sum $D^\gamma+D_N^{e^+e^-}$.

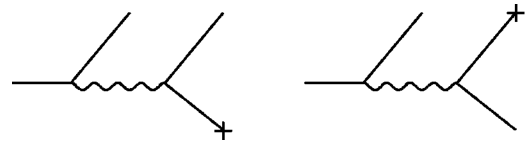

FIG. 2. Feynmann diagrams needed to calculate the electron structure function $D_S^{e+e-}(z,L)$ in perturbation theory.

The singlet structure function $D_S^{e^+e^-}$ is regular at $z=1$ and needs to be calculated only in the lowest nonvanishing order. The corresponding solution of the DGLAP equation reads [5]

$$D_S^{e^+e^-(u)}(z,L)=\frac{\alpha^2}{4\pi^2}L^2\left[\frac{2(1-z^3)}{z}+\frac{1}{2}(1-z)+(1+z)\ln z\right]\theta\left(1-z-\frac{2m}{\varepsilon}\right). \tag{11}$$

For the polarized case, the equation for $D_S^{e^+e^-(p)}$ does not exist and in order to obtain it one needs to compute the leading contribution of diagrams in Fig. 2 in kinematics when all particles move almost parallel to the parent electron. Such calculations were performed in [9] and the result is

$$D_S^{e^+e^-(p)}(z,L)=\frac{\alpha^2}{4\pi^2}L^2\left(\frac{5(1-z)}{2}+(1+z)\ln z\right)\times\theta\left(1-z-\frac{2m}{\varepsilon}\right). \tag{12}$$

The integration limits with respect to z_1 and z_2 in the master formula (3) can be found from the constraint on the Bjorken variable $\hat{x}$ for the partonic process

$$\hat{x}=\frac{-(\hat{k}_1-\hat{k}_2)^2}{2p_1(\hat{k}_1-\hat{k}_2)}=\frac{z_1yx}{z_1z_2+y-1}<1,\quad x=\frac{Q^2}{2p_1(k_1-k_2)}. \tag{13}$$

By taking into account also that $z_{1,2}<1$ and $xy=Q^2/V$, we derive, from Eq. (13),

$$1>z_2>z_{2m},\quad 1>z_1>z_{1m},\quad z_{2m}=\frac{1-y}{z_1}+\frac{Q^2}{V},$$

$$z_{1m}=\frac{V(1-y)}{V-Q^2}. \tag{14}$$

In the framework of the leading logarithmic approximation, we have to take the elastic (Born) cross section as the hard part under the integral on the right-hand side of Eq. (3)

$$\frac{d\sigma_{hard}^{\parallel,\perp(B)}}{d\,Q^2 d\,y}=\frac{d\sigma^{\parallel,\perp(B)}}{d\,Q^2}\,\delta\!\left(y-\frac{Q^2}{V}\right). \qquad (15)$$

In the case of the longitudinal polarization of the recoil proton, we have

$$\frac{d\sigma_{hard}^{\parallel(B)}}{d\,Q^2}=\frac{4\pi\alpha^2(-Q^2)}{VQ^2}\left(1-\frac{Q^2}{2V}\right)\sqrt{\frac{Q^2}{4M^2+Q^2}}G_M^2(-Q^2). \qquad (16)$$

The quantity $\alpha(-Q^2)$ on the right-hand side of Eq. (16) is the running electromagnetic constant that accounts for vacuum polarization effects

$$\alpha(q^2)=\frac{\alpha}{1-\dfrac{\alpha}{3\pi}\ln\dfrac{-q^2}{m^2}}.$$

For the transverse polarization of recoil proton, the hard part of the cross section reads

$$\frac{d\sigma_{hard}^{\perp(B)}}{d\,Q^2}=-2\frac{4\pi\alpha^2(-Q^2)}{VQ^2}\frac{M}{\sqrt{Q^2+4M^2}}\times\sqrt{1-\frac{Q^2}{V}(1+\tau)}G_E(-Q^2)G_M(-Q^2),$$

$$\tau=\frac{M^2}{V}. \qquad (17)$$

Note that in the zeroth order of perturbation theory the photon contribution into the electron structure function gives an ordinary δ function because [see also the iterative form (7)]

$$\lim_{\beta\to 0}\frac{1}{2}\beta(1-z)^{(1/2)\beta-1}=\delta(1-z). \qquad (18)$$

It is easy to see that the representation (3) reproduces the Born cross section in this case

$$\frac{d\sigma^{\parallel,\perp}}{d\,Q^2 d\,y}=\int dz_1\int dz_2\frac{1}{z_2^2}\delta(1-z_1)\delta(1-z_2)\times\frac{d\sigma^{\parallel,\perp(B)}}{d\hat{Q}^2}\delta\!\left(\hat{y}-\frac{\hat{Q}^2}{\hat{V}}\right)=\frac{d\sigma^{\parallel,\perp(B)}}{d\,Q^2}\delta\!\left(y-\frac{Q^2}{V}\right). \qquad (19)$$

III. BEYOND THE LEADING APPROXIMATION

We can improve the leading approximation for $d\sigma^{\parallel,\perp}/dQ^2dy$ given by formula (3) with $d\sigma^{\parallel,\perp(B)}/dQ^2dy$ as a hard part of the cross section under the integral. It can be done by making more precise the expression namely for this hard part

$$\frac{d\sigma_{hard}^{\parallel,\perp}}{d\,Q^2 d\,y}=\frac{d\sigma^{\parallel,\perp(B)}}{d\,Q^2 d\,y}+\frac{d\sigma^{\parallel,\perp(1)}}{d\,Q^2 d\,y}. \qquad (20)$$

The additional term on the right-hand side of Eq. (20) takes into account RC due to real and virtual photon emissions without its leading part that has already accounted for D functions. To find $d\sigma^{\parallel,\perp(1)}/d\,Q^2 d\,y$, we should calculate the corresponding cross sections of the process (1) (with virtual and soft corrections) and of the process (2), and then subtract from their sum the right-hand side of formula (3) with

$$\frac{d\sigma_{hard}^{\parallel,\perp}}{d\,Q^2 d\,y}=\frac{d\sigma^{\parallel,\perp(B)}}{d\,Q^2 d\,y},$$

which appears in the same order of perturbation theory.

We begin with calculation of the cross section of the radiative process (2) (the corresponding polarization calculations were performed earlier for deep inelastic scattering [12])

$$\frac{d\sigma^{\gamma(p)}}{d\,Q^2 d\,y}=\frac{2\pi\alpha^2(q^2)}{Vq^4}\frac{\alpha}{4\pi^2}L_{\mu\nu}^{\gamma}H_{\mu\nu}\frac{d^3k}{k_0}\frac{d^3p_2}{p_{20}}\times\delta(p_1+k_1-k_2-p_2-k), \qquad (21)$$

where $q=k_1-k_2-k=p_2-p_1$. Herein after we will be interested in the polarization-dependent parts of leptonic ($L_{\mu\nu}$) and hadronic ($H_{\mu\nu}$) tensors and assume that the degree of initial electron polarization is equal to 1. Then we have

$$H_{\mu\nu}=-iM\epsilon_{\mu\nu\lambda\rho}q_\lambda\left[-G_E(q^2)A_\rho+\frac{2[G_E(q^2)-G_M(q^2)]}{4M^2-q^2}(Ap_1)p_{1\rho}\right]G_M(q^2), \qquad (22)$$

$$L_{\mu\nu}^{\gamma}=-2i\epsilon_{\mu\nu\lambda\rho}q_\lambda[k_{1\rho}R_t+k_{2\rho}R_s],$$

$$R_t=\frac{u+t}{st}-2m^2\left(\frac{1}{s^2}+\frac{1}{t^2}\right),\quad R_s=\frac{u+s}{st}-2m^2\frac{s_t}{ut^2},$$

$$s_t=\frac{-u(u+Vy)}{u+V}, \qquad (23)$$

where A is the 4-vector of recoil proton polarization and we use the following notation for kinematic invariants:

$$u=(k_1-k_2)^2,\quad s=2kk_2,\quad t=-2kk_1,$$

$$q^2=u+s+t,\quad Q^2=-u.$$

It is convenient to express the recoil proton polarization 4-vector A in terms of the particle 4-momenta and Lorentz invariants. Below we use the following parametrization for $A^\parallel$ and $A^\perp$:

$$A_\mu^{\parallel}=\frac{2M^2q_\mu-q^2p_{2\mu}}{MQ_{\parallel}},\quad Q_{\parallel}=\sqrt{-q^2(4M^2-q^2)},\tag{24}$$

$$A_\mu^{\perp}=\frac{2[2M^2k_1q+q^2k_1p_1]p_{2\mu}-2[2M^2k_1q-q^2k_1p_2]p_{1\mu}+q^2(q^2-4M^2)k_{1\mu}}{2Q_{\perp}},$$

$$Q_{\perp}=\sqrt{q^2[q^2M^2(k_1p_1+k_1p_2)^2+(2M^2k_1q-q^2k_1p_2)(2M^2k_1q+q^2k_1p_1)]},\tag{25}$$

$$2k_1p_2=V+u+t,\quad 2k_1q=u+t.$$

It is easy to verify that the 4-vector $A^{\parallel}$ in the rest frame of the recoil proton has components $(0,\vec{n})$, where 3-vector $\vec{n}$ has an orientation of the recoil proton 3-momentum in a laboratory system. One can verify also the orthogonality conditions, $A^{\perp}A^{\parallel}=0$, and the following relations valid in the rest frame of the recoil proton:

$$A^{\perp}=(0,\vec{n}_{\perp}),\quad \vec{n}_{\perp}^2=1,\quad \vec{n}\vec{n}_{\perp}=0,$$

where the 3-vector $\vec{n}_{\perp}$ is within the plane $(\vec{k}_1,\vec{p}_2)$ in the laboratory system.

For longitudinal polarization, the contraction of leptonic and hadronic tensors yields

$$\frac{L^{\gamma}_{\mu\nu}H_{\mu\nu}}{q^4}=-\frac{2m^2}{s^2}(q_s^2+2V)F(q_s^2)-\frac{2m^2}{t^2}(u+2V)\times\left(1+\frac{s_tq_t^2}{u^2}\right)F(q_t^2)+\left\{\frac{1}{tu}[(u^2+q^4)(u+2V)-2q^2(q^2-q_t^2)(u+V)]+\frac{1}{sq_s^2}[(q^4+u^2)\times(q_s^2+2V)-2q^2V(q^2-q_s^2)]\right\}\frac{F(q^2)}{q^2-u},\tag{26}$$

where

$$q_t^2=u+s_t=\frac{uV(1-y)}{u+V},\quad q_s^2=u+t_s=\frac{uV}{V(1-y)-u},$$

$$t_s=\frac{u(u+Vy)}{V(1-y)-u},$$

$$F(q^2)=-G_M^2(q^2)\frac{1}{q^2}\sqrt{\frac{-q^2}{4M^2-q^2}}.$$

The physical meaning of quantities q_t^2 and q_s^2 is as follows: q_t^2 and q_s^2 are the values of q^2 in the cases of the initial-state and final-state collinear radiation, respectively. When writing the formula (26), we took into account the fact that the terms containing the electron mass squared contribute only in collinear kinematics.

To separate the contribution into the right-hand side of Eq. (26) due to collinear radiation for the polelike terms, we apply the operations $\hat{P}_t$ and $\hat{P}_s$,

$$\frac{1}{t}f(q^2,u,t,s)=\frac{1}{t}(1-\hat{P}_t+\hat{P}_t)f(q^2,u,t,s),$$

$$\hat{P}_tf(q^2,u,s,t)=f(q_t^2,u,s_t,0)$$

for an arbitrary nonsingular function at $t\to0$ and similarly for $1/s$ terms. Therefore, we can rewrite the right-hand side of Eq. (26) in the form

$$\left\{-\frac{2m^2}{s^2}(q_s^2+2V)\hat{P}_s-\frac{2m^2}{t^2}(u+2V)\left(1+\frac{s_tq_t^2}{u}\right)\hat{P}_t\right\}F(q^2)+\left\{\frac{(u+2V)(u^2+q_t^4)}{ut}\hat{P}_t+\frac{(q_s^2+2V)(u^2+q_s^4)}{q_s^2s}\hat{P}_s+\frac{1-\hat{P}_t}{ut}[(u+2V)(u^2+q^4)-2q^2(q^2-q_t^2)(u+V)]+\frac{1-\hat{P}_s}{q_s^2s}[(q_s^2+2V)(u^2+q^4)-2q^2V(q^2-q_s^2)]\right\}\frac{F(q^2)}{q^2-u}.\tag{27}$$

For the case of transverse polarization, the contraction of leptonic and hadronic tensors has a more complex form,

$$\frac{1}{q^4}L^{\gamma}_{\mu\nu}H_{\mu\nu}=([q^2(u+t+2V)^2+(4M^2-q^2)(u+t)^2]R_t+\{q^2(u+t+2V)[t-q^2+2V(1-y)]+(4M^2-q^2)(uq^2-st)\}R_s)\frac{G_E(q^2)G_M(q^2)}{q^4}\times\sqrt{\frac{-q^2M^2}{(4M^2-q^2)[-q^2V(V+u+t)-M^2(u+t)^2]}}.\tag{28}$$

The expression in the round brackets on the right-hand side of Eq. (28) can be rewritten in the form suitable for the photon angular integration as follows:

$$-2[q^2Vy+4M^2(q^2+u)]-\frac{2m^2}{s^2}4V^2q_s^2K_s\hat{P}_s-\frac{2m^2}{t^2}4V^2q_t^2\left(1+\frac{s_tq_t^2}{u^2}\right)K_t\hat{P}_t+\frac{1}{t}\left[\frac{4V^2q_t^2(u^2+q_t^4)}{u(q_t^2-u)}\times K_t\hat{P}_t+(1-\hat{P}_t)\frac{q^2}{u(q^2-u)}[4V^2(u^2+q^4)K_q-2q^2\times(q^2-q_t^2)(u+2V)(u+V)]\right]+\frac{1}{s}\left[\frac{4V^2(u^2+q_s^4)}{(q_s^2-u)}K_s\hat{P}_s+(1-\hat{P}_s)\frac{q^2V}{q_s^2(q^2-u)}[4V(u^2+q^4)K_s-2V(q^2-q_s^2)\times(2Vq^2-u^2)]\right],$$

$$K_s=1+\frac{q_s^2}{V}(1+\tau),\quad K_t=1+\frac{u}{V}+\frac{u^2\tau}{Vq_t^2},$$

$$K_q=1+\frac{u}{V}+\frac{u^2\tau}{Vq^2}. \tag{29}$$

To perform photon angular integration, we choose the system $\vec{k}_1+\vec{p}_1-\vec{k}_2=0$. In this system the energies of particles are

$$k_0=\frac{a}{2\sqrt{R}},\quad k_{10}=\frac{u+V}{2\sqrt{R}},\quad k_{20}=\frac{V(1-y)-u}{2\sqrt{R}},$$

$$p_{10}=\frac{2M^2+Vy}{2\sqrt{R}},\quad p_{20}=\frac{R+M^2}{2\sqrt{R}},$$

$$a=u+Vy,\quad R=a+M^2. \tag{30}$$

Taking the Z-axis along the initial proton 3-momentum in the chosen system, we also have

$$c_k=\cos\theta_k=\frac{2M^2-2p_{10}p_{20}-q^2}{2|\vec{p}_1||\vec{p}_2|},$$

$$c_2=\cos\theta_2=\frac{2k_{20}p_{10}-V(1-y)}{2|\vec{p}_1||\vec{k}_2|},$$

$$c_1=\cos\theta_1=\frac{2k_{10}p_{10}-V}{2|\vec{p}_1||\vec{k}_1|},$$

$$|\vec{p}_1|=\frac{\sqrt{V^2y^2-4uM^2}}{2\sqrt{R}},\quad |\vec{p}_2|=k_0, \tag{31}$$

where $\theta_1(\theta_2)$ is the polar angle of the initial (scattered) electron and θ_k is the photon polar angle. Besides Eqs. (30) and (31), we will use the relation

$$\frac{d^3k}{k_0}\frac{d^3p_2}{p_{20}}\delta(k_1+p_1-k_2-k-p_2)=\frac{a}{2R}d\varphi\, d\cos\theta_k. \tag{32}$$

Let us concentrate on the case of longitudinal polarization of the recoil proton. For the terms containing m^2/s^2, m^2/t^2, $\hat{P}_t/t$ and $\hat{P}_s/s$, we can use the following formulas:

$$\int\frac{m^2d\varphi d\cos\theta_k}{2\pi s^2}=\int\frac{m^2d\varphi d\cos\theta_k}{2\pi t^2}=\frac{2R}{a^2},$$

$$\int\frac{d\varphi d\cos\theta_k}{2\pi s}=\frac{2R}{a(V(1-y)-u)}(L_s+L),$$

$$\int\frac{d\varphi d\cos\theta_k}{2\pi(-t)}=\frac{2R}{a(u+V)}(L_t+L),$$

$$L_s=\ln\frac{(V(1-y)-u)^2}{-uR},\quad L_t=\ln\frac{(V+u)^2}{-uR}. \tag{33}$$

Terms which contain $(1-\hat{P}_t)$, $(1-\hat{P}_s)$ operators can be integrated over the azimuthal angle, while retaining integration with respect to q^2 using the transformation $d\cos\theta_k=dq^2/2|\vec{p}_1||\vec{p}_2|$,

$$\int\frac{d\varphi}{2\pi s2|\vec{p}_1||\vec{p}_2|}=\frac{2R}{a|q^2-q_s^2|[V(1-y)-u]},$$

$$\int\frac{d\varphi}{2\pi(-t)2|\vec{p}_1||\vec{p}_2|}=\frac{2R}{a|q^2-q_t^2|(V+u)}. \tag{34}$$

The limits of q^2 integration in this case can be derived from the restriction on $\cos\theta_k$ in the chosen system, $|\cos\theta_k|<1$. This restriction leads to the relation

$$q_-^2<q^2<q_+^2,$$

$$q_\pm^2=\frac{1}{2R}[2uM^2-Vy(u+Vy)\pm(u+Vy)\sqrt{V^2y^2-4uM^2}]. \tag{35}$$

By using Eqs. (33), (34), and (35), we can write the cross section of the radiative process (2) for the longitudinal polarization of recoil proton as follows:

$$\frac{d\sigma^{\parallel\gamma}}{dQ^2dy}=\frac{2\alpha}{V}\Bigg\{-\frac{q_s^2+2V}{u+Vy}\hat{P}_s-\frac{(u+2V)(u^2+s_tq_t^2)}{u^2(u+Vy)}\hat{P}_t-[1+L_t+(L-1)]\frac{(u+2V)(u^2+q_t^4)}{2u(u+V)(q_t^2-u)}\hat{P}_t+[1+L_s+(L-1)]\frac{(q_s^2+2V)(u^2+q_s^4)}{2q_s^2(V(1-y)-u)(q_s^2-u)}\hat{P}_s+\int_{q_-^2}^{q_+^2}\Bigg[-\frac{dq^2}{|q^2-q_t^2|}(1-\hat{P}_t)\times\frac{(u+2V)(u^2+q^4)-2q^2(q^2-q_t^2)(u+V)}{2u(u+V)(q^2-u)}+\frac{dq^2}{|q^2-q_s^2|}(1-\hat{P}_s)\times\frac{(q_s^2+2V)(u^2+q^4)-2q^2V(q^2-q_s^2)}{2q_s^2[V(1-y)-u](q^2-u)}\Bigg]\Bigg\}\times\alpha^2(q^2)F(q^2)\,\theta\left(y+\frac{u}{V}-\frac{2M\Delta\varepsilon}{V}\right). \quad (36)$$

The θ function appears on the right side of Eq. (36) due to the restriction on the photon hardness in the radiative process (2)

$$k_0=\frac{u+Vy}{2\sqrt{M^2+u+Vy}}>\Delta\varepsilon\rightarrow y>-\frac{u}{V}+\frac{2M\Delta\varepsilon}{V}, \quad (37)$$

where $\Delta\varepsilon$ is the minimal photon energy in the chosen coordinate system.

To be complete, we should also take into account RC due to virtual and soft (with the energy smaller than $\Delta\varepsilon$) photon emission to the cross section of the elastic process (1). It can be written as (see, for example, [6])

$$\frac{d\sigma^{\parallel(V+S)}}{dQ^2dy}=\frac{4\pi\alpha^2(-Q^2)}{V}\left(1-\frac{Q^2}{2V}\right)F(-Q^2)\frac{\alpha}{2\pi}\Bigg[2(L-1)\times\left(\ln\frac{4M^2(\Delta\varepsilon)^2}{V(u+V)}+\frac{3}{2}\right)-1-\frac{\pi^2}{3}-\ln^2\frac{u+V}{V}-2f\left(\frac{u+V+u\tau}{u+V}\right)\Bigg]\delta\left(y-\frac{Q^2}{V}\right),$$

$$f(y)=\int_0^y\frac{dx}{x}\ln(1-x). \quad (38)$$

Therefore, the sum of the cross sections of the processes (1) and (2) is defined by the formula

$$\frac{d\sigma^{\parallel(B)}}{dQ^2dy}+\frac{d\sigma^{\parallel\gamma}}{dQ^2dy}+\frac{d\sigma^{\parallel(S+V)}}{dQ^2dy}. \quad (39)$$

To include the hard cross section into the electron structure function representation (3) in the form (39) and to get rid of the double counting, we must remove from the sum (39) the contribution which arises in the representation (3) in the first order with respect to the fine structure constant α at

$$\frac{d\sigma^{\parallel}_{hard}}{dQ^2dy}=\frac{d\sigma^{\parallel(B)}}{dQ^2dy}.$$

The procedure for deriving this contribution is described in [6]. We can verify that it equals to

$$\frac{2\alpha}{V}\Bigg\{(L-1)\Bigg[-\frac{(u+2V)(u^2+q_t^4)}{2u(u+V)(q_t^2-u)}\hat{P}_t+\frac{(q_s^2+2V)(u^2+q_s^4)}{2q_s^2(V(1-y)-u)(q_s^2-u)}\hat{P}_s\Bigg]\alpha^2(q^2)F(q^2)\,\theta\times\left(y+\frac{u}{V}-\frac{2M\Delta\varepsilon}{V}\right)+2(L-1)\left(\ln\frac{4M^2(\Delta\varepsilon)^2}{V(u+V)}+\frac{3}{2}\right)\times\left(1-\frac{Q^2}{2V}\right)\alpha^2(-Q^2)F(-Q^2)\,\delta\left(y+\frac{u}{V}\right)\Bigg\}. \quad (40)$$

Thus, we can write the final result for the $d\sigma^{\parallel}_{hard}/dQ^2dy$ in the following very compact form:

$$\frac{d\sigma^{\parallel}_{hard}}{dQ^2dy}=\frac{d\sigma^{\parallel(B)}}{dQ^2dy}\Bigg\{1+\frac{\alpha}{2\pi}\Bigg[-1-\frac{\pi^2}{3}-\ln^2\frac{u+V}{V}-2f\left(\frac{u+V+u\tau}{u+V}\right)\Bigg]\Bigg\}+\frac{2\alpha}{V}\Bigg\{\frac{(u+2V)(q_t^2-u)}{2u(u+V)}\hat{P}_t+\frac{(q_s^2+2V)(q_s^2-u)}{2uV}\hat{P}_s+\mathrm{P}\int_{q_-^2}^{q_+^2}\frac{dq^2}{q^2-u}\Bigg[\frac{1}{|q^2-q_s^2|}\times(1-\hat{P}_s)\frac{(q_s^2+2V)(u^2+q^4)-2q^2V(q^2-q_s^2)}{2q_s^2(V(1-y)-u)}-\frac{1}{|q^2-q_t^2|}(1-\hat{P}_t)\times\frac{(u+2V)(u^2+q^4)-2q^2(u+V)(q^2-q_t^2)}{2u(V+u)}\Bigg]\Bigg\}\times\alpha^2(q^2)F(q^2)\,\theta\left(y+\frac{u}{V}\right), \quad (41)$$

where P stands for the principal value integration. When writing the last formula, we used the following relations:

$$\mathrm{P}\int_{q_-^2}^{q_+^2}\frac{d\,q^2[f(q^2)-f(q_t^2)]}{|q^2-q_t^2|(q^2-u)}=\frac{f(q_t^2)}{q_t^2-u}L_t+\int_{q_-^2}^{q_+^2}\frac{d\,q^2}{|q^2-q_t^2|}\times\left(\frac{f(q^2)}{q^2-u}-\frac{f(q_t^2)}{q_t^2-u}\right), \quad (42)$$

$$\mathrm{P}\int_{q_-^2}^{q_+^2}\frac{d\,q^2[f(q^2)-f(q_s^2)]}{|q^2-q_s^2|(q^2-u)}=\frac{f(q_s^2)}{q_s^2-u}L_s+\int_{q_-^2}^{q_+^2}\frac{d\,q^2}{|q^2-q_s^2|}\times\left(\frac{f(q^2)}{q^2-u}-\frac{f(q_s^2)}{q_s^2-u}\right), \quad (43)$$

where the symbol P indicates how we shall integrate the unphysical singularity at $q^2=u$. These relations allow us to see that infrared singularities of separate terms in $d\sigma^{\parallel(1)}/dQ^2dy$ exactly cancel each other. That is why we omitted the term $-2M\Delta\epsilon/V$ from the argument of the θ function on the right side of Eq. (41). For numerical calculations the principle value integration can be understood as

$$\mathrm{P}\int_{q_-^2}^{q_+^2}\frac{d\,q^2}{q^2-u}F(q^2)=\int_{q_-^2}^{q_+^2}\frac{d\,q^2}{q^2-u}[F(q^2)-F(u)]+F(u)\log\frac{q_+^2-u}{q_-^2-u}.$$

The hard part of the cross section in the case of transverse polarization of the recoil proton can be derived in full analogy with the above. The main difference is caused by the fact that the vector of the transverse polarization has a complicated dependence on the photon azimuthal angle ϕ and therefore even ϕ integration becomes nontrivial. The straightforward calculations give

$$\frac{d\sigma^{\perp}_{hard}}{d\,Q^2d\,y}=\frac{d\sigma^{\perp(B)}}{d\,Q^2d\,y}\left\{1+\frac{\alpha}{2\pi}\left[-1-\frac{\pi^2}{3}-\ln^2\frac{u+V}{V}-2f\left(\frac{u+V+u\tau}{u+V}\right)\right]\right\}+\frac{2\alpha}{V}\left\{\left[\frac{2(q_t^2-u)V}{u(u+V)}+\frac{2V(u^2+q_t^4)}{u^2(u+Vy)}L_t\right]\frac{K_t\hat{P}_t}{q_t^2}+\left[\frac{2(q_s^2-u)}{u}+\frac{2V(u^2+q_s^4)}{uq_s^2(u+Vy)}L_s\right]\frac{K_s\hat{P}_s}{q_s^2}+\int_{q_-^2}^{q_+^2}\frac{d\,q^2}{\sqrt{V^2y^2-4uM^2}}\int_0^{2\pi}\frac{d\varphi}{2\pi}\left[\frac{[-yq^2-4\tau(u+q^2)]}{q^4}+\frac{1-\hat{P}_t}{t}\left(\frac{2V(u^2+q^4)}{uq^2(q^2-u)}K_q-\frac{(u+2V)(u+V)(q^2-q_t^2)}{uV(q^2-u)}\right)+\frac{1-\hat{P}_s}{s}\left(\frac{2V(u^2+q^4)}{q_s^2q^2(q^2-u)}K_s+\frac{(u^2-2q^2V)(q^2-q_s^2)}{q_s^2q^2(q^2-u)}\right)\right]\right\}\alpha^2(q^2)\sqrt{\frac{M^2}{4M^2-q^2}}\left(1+\frac{u+t}{V}+\frac{(u+t)^2\tau}{Vq^2}\right)^{-1/2}G_E(q^2)G_M(q^2)\,\theta\left(y+\frac{u}{V}\right). \quad (44)$$

For invariants s and t on the right side of Eq. (44), we can neglect the electron mass and use here the simplified expressions

$$s=c_{2i}-s_i\cos\varphi,\qquad -t=c_{1i}-s_i\cos\varphi,$$

$$c_{1i}=2k_0k_{10}[1-\cos\theta_1\cos\theta_k],$$

$$c_{2i}=2k_0k_{20}[1-\cos\theta_2\cos\theta_k], \quad (45)$$

$$s_i=2k_0k_{10}\sin\theta_1\sin\theta_k=2k_0k_{20}\sin\theta_2\sin\theta_k.$$

The integrals over ϕ can be calculated analytically in terms of elliptic functions $\mathcal{K}$ and Π

$$\int_0^{2\pi}\frac{d\varphi}{2\pi}\left(1+\frac{u+t}{V}+\frac{(u+t)^2\tau}{Vq^2}\right)^{-1/2}=J_0=\frac{2}{\pi\sqrt{X}}\mathcal{K}(\kappa),$$

$$\int_0^{2\pi}\frac{d\varphi}{2\pi t}\left(1+\frac{u+t}{V}+\frac{(u+t)^2\tau}{Vq^2}\right)^{-1/2}=\frac{J_t}{|q^2-q_t^2|}=-\frac{B_t(1+b_t)\sqrt{\lambda y}}{2(V+u)|q^2-q_t^2|\sqrt{X}}\times\left(\frac{2}{\pi}\sqrt{1-b_{1t}}\mathcal{K}(\kappa)+\frac{B_t}{b_{1t}y}\frac{1-\Lambda(\epsilon_t,\kappa)}{\sqrt{1-\kappa^2/b_{1t}}}\right), \quad (46)$$

$$\int_0^{2\pi}\frac{d\varphi}{2\pi s}\left(1+\frac{u+t}{V}+\frac{(u+t)^2\tau}{Vq^2}\right)^{-1/2}=\frac{J_s}{|q^2-q_s^2|}=\frac{B_s(1+b_s)\sqrt{\lambda y}}{2(V-a)|q^2-q_s^2|\sqrt{X}}\times\left(\frac{2}{\pi}\sqrt{1-b_{1s}}\mathcal{K}(\kappa)+\frac{B_s}{b_{1s}y}\frac{1-\Lambda(\epsilon_s,\kappa)}{\sqrt{1-\kappa^2/b_{1s}}}\right),$$

where

$$X=(1+x_+)(1-x_-)\frac{M^2s_i^2}{-q^2V^2},$$

$$\kappa^2=\frac{2(x_+-x_-)}{(1+x_+)(1-x_-)},$$

$$\bar{y}=\frac{-x_-+1}{-x_--1},$$

$$\begin{aligned}2M^2s_ix_\pm&=2M^2(-q^2+c_{2i})-Vq^2(1\pm\sqrt{1-4M^2/q^2})\\&=2M^2(-u+c_{1i})-Vq^2(1\pm\sqrt{1-4M^2/q^2})\end{aligned} \tag{47}$$

$$b_{1s,t}=\frac{1+\bar{y}-b_{s,t}+b_{s,t}\bar{y}}{\bar{y}(1+b_{s,t})},\quad b_{s,t}=\frac{c_{2,1i}}{s_i},$$

$$B_{s,t}=b_{1s,t}\bar{y}+1-\bar{y}.$$

The function $\Lambda(\epsilon,\kappa)$ ($\epsilon=\arcsin[(1-b_1)/(1-\kappa^2)],\epsilon_{t,s}=\epsilon(b_1\to b_{1+,s})$) is nonsingular Heuman's Lambda function varying from 0 to 1 (see [13] for details and exact definitions). It is related to complete elliptic integral $\Pi(b_1,\kappa)$ of the third kind

$$\frac{2}{\pi}\Pi(b_1,\kappa)=\frac{1-\Lambda(\epsilon,\kappa)}{\sqrt{1-b_1}\sqrt{1-\kappa^2/b_1}}+\frac{2}{\pi}\mathcal{K}(\kappa). \tag{48}$$

For $\epsilon\to0$ (or $b_1\to1$) this function goes to zero. In the last formula the singular behavior of $\Pi(b_1,\kappa)$ for $b_1\to1$ is expressed explicitly in the first term. This limit corresponds to collinear radiation:

$$\sqrt{1-b_{1t}}=\frac{u+V}{(1+b_t)s_i\sqrt{\bar{y}\lambda}}|q^2-q_t^2|,$$

$$\sqrt{1-b_{1s}}=\frac{V-a}{(1+b_s)s_i\sqrt{\bar{y}\lambda}}|q^2-q_s^2|, \tag{49}$$

where $\lambda=y^2V^2-4M^2u$. As a result of substituting Eqs. (46)–(48) into the formula for the hard cross section (44) we arrive at the same structure of singularities as in the longitudinal case (36). In the collinear limit $q^2\to q_{t,s}^2$, we have

$$\bar{y}X(1-\kappa^2/b_{1t,s})\to K_{t,s}\quad b_{t,s},b_{1t,s},B_{t,s}\to1,\quad \Lambda(\epsilon,\kappa)\to0.$$

These limiting formulas allow us to use relations (42) and (43) to write the final expression for hard cross section in such a form that provides explicit cancellation of infrared divergence in the same way as for the longitudinal polarization.

Combining all results together, we obtain the final formula for the cross section in the transverse polarized case:

$$\begin{aligned}\frac{d\sigma^\perp_{hard}}{dQ^2dy}=&\frac{d\sigma^{\perp(B)}}{dQ^2dy}\Bigg\{1+\frac{\alpha}{2\pi}\Bigg[-1-\frac{\pi^2}{3}-\ln^2\frac{u+V}{V}\\&-2f\left(\frac{u+V+u\tau}{u+V}\right)\Bigg]\Bigg\}+\frac{2\alpha}{V}\Bigg\{\frac{2(q_t^2-u)V}{u(u+V)}\frac{K_t\hat{P}_t}{q_t^2}\\&+\frac{2(q_s^2-u)}{u}\frac{K_s\hat{P}_s}{q_s^2}+P\int_{q_-^2}^{q_+^2}\frac{dq^2}{\sqrt{\lambda}(q^2-u)}\\&\times\Bigg[\frac{[-yq^2-4\tau(u+q^2)]}{q^4}(q^2-u)J_0+\frac{1-\hat{P}_t}{|q^2-q_t^2|}J_t\\&\times\left(\frac{2V(u^2+q^4)}{uq^2}K_q-\frac{(u+2V)(u+V)(q^2-q_t^2)}{uV}\right)\\&+\frac{1-\hat{P}_s}{|q^2-q_s^2|}J_s\Bigg(\frac{2V(u^2+q^4)}{q_s^2q^2}K_s\\&+\frac{(u^2-2q^2V)(q^2-q_s^2)}{q_s^2q^2}\Bigg)\Bigg]\Bigg\}\alpha^2(q^2)\\&\times\sqrt{\frac{M^2}{4M^2-q^2}}G_E(q^2)G_M(q^2)\,\theta\left(y+\frac{u}{V}\right).\end{aligned} \tag{50}$$

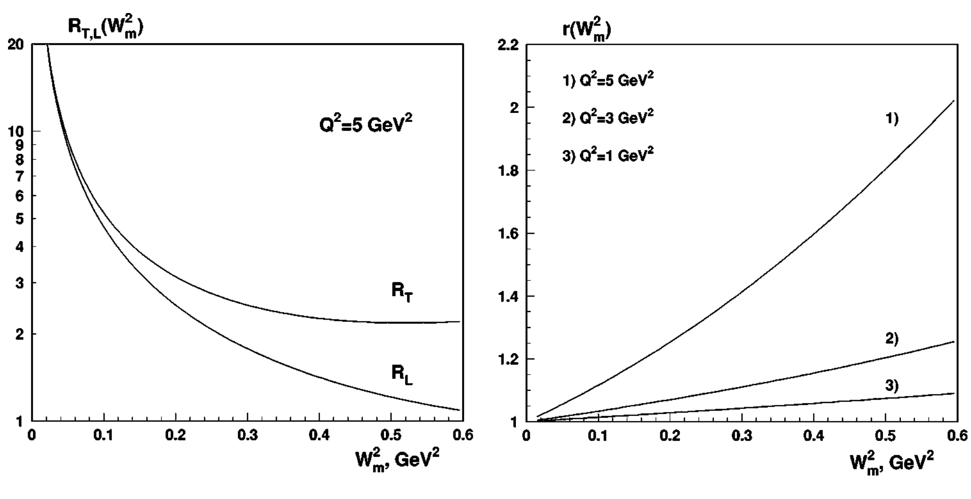

FIG. 3. Longitudinal and transverse polarization parts of cross sections normalized to Born ones (left plot) and their ratios (right plot) [see Eq. (52) for exact definitions] as a function of missing mass squared for beam energy 4.26 GeV ($V=8$ GeV^2).

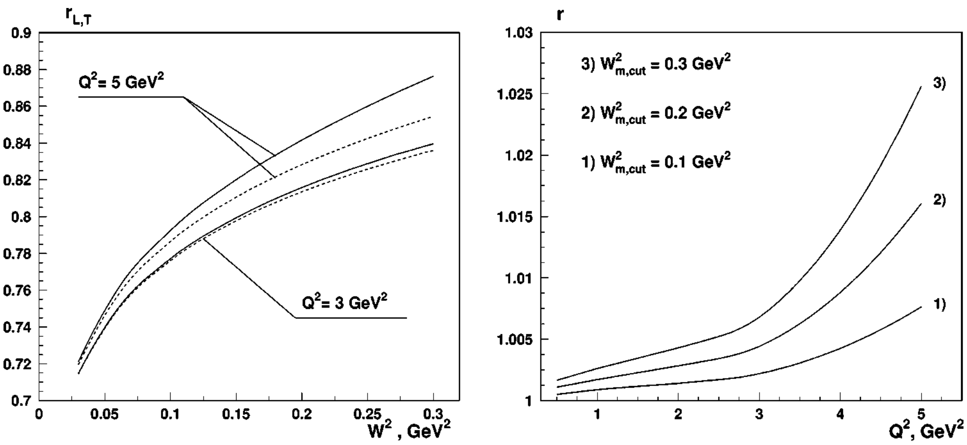

FIG. 4. Radiative correction to recoil polarization rations $r_{T,L}$ (left plot) and r (right plot) (52) within the kinematical conditions of Jefferson Lab, as a function of Q^2 and value of a cut on missing mass for beam energy 4.26 GeV ($V=8$ GeV^2). Solid (dashed) line on the left plot shows $r_T(r_L)$.

The theoretical formula for the ratio of longitudinal and transverse polarizations of the recoil proton that was measured in recent experiments [2,3] is defined by the ratio of the right-hand side of Eq. (3) for longitudinal polarization [with Eq. (41) as the hard cross section under the integral sign] and for the transverse polarization [with Eq. (44) as the hard cross section]. This high precision formula takes into account model-independent RC with all the leading and the main part of the next-to-leading corrections, and has accuracy at the level of 0.1%.

IV. NUMERICAL ANALYSIS

The ratio of proton elastic form factors G_e/G_m measured experimentally [2,3] is related to the ratio of recoiled proton polarization components. At the Born level (i.e., without RC) the ratio of polarizations is defined by the ratio of a spin-dependent cross section given by Eqs. (16) and (17):

$$\frac{P_T}{P_L}=\frac{\sigma_T^0}{\sigma_L^0}. \tag{51}$$

The photon spectrum can be defined as a function of the missing mass $W_m^2=yV-Q^2$ (either y or photon energy in the chosen frame E_γ) of the observed cross section $\sigma_{T,L}(W_m^2)$ defined by the master equation (3). An integral over y gives a radiative correction to recoil polarizations and to their ratio. Let us define the following quantities:

$$R_{T,L}(W_m^2)=\frac{\sigma_{T,L}(W_m^2)}{\sigma_{T,L}^0}, \quad r(W_m^2)=\frac{R_T(W_m^2)}{R_L(W_m^2)},$$

$$r_{T,L}=\int\frac{dW_m^2}{V}R_{T,L}(W_m^2), \quad r=\frac{r_T}{r_L}. \tag{52}$$

In Fig. 3 the $R_{T,L}$ as a function of missing mass is presented. For very small values of missing mass or alternatively for $y\to Q^2/V$ the cross sections reproduce the δ-function behavior. In the limit (18), there are three delta functions (from D^u, D^p, and the y dependence of the Born cross section) and only double integration. So we have behavior as in Eq. (18) in this limit. Only the factorizable part is important here, so both longitudinal and transverse $R's$ are practically the same. For larger values of W_m^2 (or y) the nonfactorized part contribution becomes important. It can be seen from Fig. 3(b), where ratios of these spectra are presented.

Figure 4 presents the results integrated over $dy=dW_m^2/V$. This integration has to be performed up to some specific values of a cut on the missing mass which is defined by experimental conditions. Using the hard cut leads to negative values of RC (or $r_{T,L}$ becomes less than one), because the contribution of loops, which is usually negative, dominates in this case. If the positive contribution of hard photon radiation is allowed by using less stringent cuts, the radiative correction to the polarized parts of cross section goes up and can exceed several tens of percents. The right plot in Fig. 4 gives a radiative correction factor to the polarization ratio or the measured ratio of form factors. One can see that the radiative correction to it is rising not only with the increasing value of the cut but also with increasing Q^2. Within the kinematical conditions of Jefferson Lab, the radiative correction is at the level of several percent or smaller if the hard cut on missing mass (or missing energy) is used.

V. DISCUSSION AND CONCLUSION

In this paper we calculated radiative corrections to observable quantities in elastic electron-proton scattering where polarization of the final proton is measured. An observable cross section of this process has to include QED loop effects and contributions of radiation of real photons and electron-positron pair creation from the lepton line. In this paper a method of structure functions is applied for this calculation. Within this approach, it is possible to calculate the contributions of leading and main part of next-to-leading order RC in all orders of perturbation theory. Obtained explicit formulas are free from infrared divergence and can be used for straightforward numerical analysis. This numerical analysis was done for the kinematic conditions of current and future experiments at Jefferson Lab. Specific values of radiative correction factors were calculated. It was shown that a radiative correction to the observable ratio of recoil polarizations is at the percent level.

We note that the problem was solved for the case when the kinematical variable Q^2 is reconstructed via the electron momentum measured. There is also a possibility that this variable is calculated using the measurement of the final proton momentum. This case requires another treatment, see Ref. [14]. Also the present calculation does not include effects due to two-photon coupling to the proton.

The target considered in this paper is a proton; however, the results can be straightforwardly generalized to the case when a nuclear target is used instead. In this case the effects of Fermi motion and the finite momentum of the spectator nucleon system have to be taken into account.

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