The pion cloud of the nucleon: Facts and popular fantasies

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• Dispersion relations: Theoretical framework
• Discussion of the spectral functions
• Results for space- and time-like ffs & the pion cloud
• Summary and outlook

with: Maxim A. Belushkin, Hans-Werner Hammer
Introduction
REMARKS & DISCLAIMER

• Analyze nucleon em ffs using dispersion theory w/ input from
  chiral perturbation theory, draw conclusions about the pion cloud

• What can we say (or not) about the pion cloud of the nucleon?

  ★ concept originated soon after Yukawas proposal of the pion
    Heisenberg, Wentzel, Pauli, Kemmer, . . .

  ★ in general, not a well-defined concept → requires some modelling

  ★ pion cloud = pion loops in CHPT?

    yes, but this clearly shows the limitations of the concept

    In general, any observable receives contributions from
    pion loops and short-distance operators → reshuffling
    since only physical observables are RG invariant

    \[ \frac{d}{d\lambda} \mathcal{O}(\lambda) = 0 \]

    ⇒ let us analyze this in more detail
LOOPS vs CONTACT OPERATORS: AN EXAMPLE


- the isovector Dirac radius of the proton at third order in the chiral expansion

$$
\langle r^2 \rangle^V_1 = \left( 0.61 - (0.47 \text{ GeV}^{-2}) \tilde{d}(\lambda) + 0.47 \log \frac{\lambda}{1 \text{ GeV}} \right) \text{ fm}^2
$$

- dimension-3 LEC $\tilde{d}$ parameterizes the nucleon “core”
- pion loops give const + chiral logarithm (at fixed pion mass)
- empirical value for pairs of $(\lambda[\text{GeV}], \tilde{d}(\lambda)[\text{GeV}^{-2}])$, e.g.

$$(1.0, +0.06) , \quad (0.943, 0.0) , \quad (0.6, -0.46)$$

$\Rightarrow$ the “core” contribution is only positive for $\lambda > 943$ MeV

$\Rightarrow$ at odds with naive expectations

- this is an extreme case, but it nicely illustrates the point
WHY DISPERSION RELATIONS for the NUCLEON FFs?

- Model-independent approach → important non-perturbative tool to analyze data
- Dispersion relations are based on fundamental principles: unitarity & analyticity
- Connect data from small to large momentum transfer
  as well as time- and space-like data
- Allow for a simultaneous analysis of all four em form factors
- Spectral functions encode perturbative and non-perturbative physics
  e.g. vector meson couplings, multi-meson continua, pion cloud, . . .
- Spectral functions also encode information on the strangeness vector current
  → sea-quark dynamics, strange matrix elements
- Allow to extract nucleon electric and magnetic radii
- Can be matched to chiral perturbation theory
Theoretical framework
BASIC DEFINITIONS

• Nucleon matrix elements of the em vector current $J^I_\mu$

\[
\langle N(p')|J^I_\mu|N(p)\rangle = \bar{u}(p') \left[ F^I_1(t) \gamma_\mu + i \frac{F^I_2(t)}{2m} \sigma_{\mu\nu} q^\nu \right] u(p)
\]

★ isospin $I = S, V$ (isoScalar, isoVector)

★ four-momentum transfer $t \equiv q^2 = (p' - p)^2 \equiv -Q^2$

★ $F_1 = \text{Dirac form factor, } F_2 = \text{Pauli form factor}$

★ Normalizations: $F^V_1(0) = F^S_1(0) = 1/2, F^{S,V}_2(0) = (\kappa_p \pm \kappa_n)/2$

★ Sachs form factors: $G_E = F_1 + \frac{t}{4m^2} F_2, \quad G_M = F_1 + F_2$

★ Nucleon radii: $F(t) = F(0) \left[ 1 + t \langle r^2 \rangle / 6 + \ldots \right]$ [except for the neutron charge ff]
The form factors have cuts in the interval \([t_n, \infty[\) \((n = 0, 1, 2, \ldots)\) and also poles

\[\Rightarrow\] Dispersion relations for \(F_i(t)\) \((i = 1, 2)\):

\[F_i(t) = \frac{1}{\pi} \int_{t_0}^{\infty} dt' \frac{\text{Im} F_i(t')}{t' - t}\]

• no subtractions
  [only proven in perturbation theory]

• suppression of higher mass states

• central objects: spectral functions
  \[\text{Im} F_i(t)\]
  – cuts \(\wedge\) multi-meson continua
  – poles \(\wedge\) vector mesons
SPECTRAL FUNCTIONS – GENERALITIES

- Spectral decomposition:

\[
\text{Im} \langle \bar{N}(p')N(p)\mid J^I_\mu \mid 0 \rangle \sim \sum_n \langle \bar{N}(p')N(p)\mid n \rangle \langle n \mid J^I_\mu \mid 0 \rangle \Rightarrow \text{Im} F
\]

★ on-shell intermediate states
★ generates imaginary part
★ accessible physical states

- *Isoscalar* intermediate states: \(3\pi, 5\pi, \ldots, K\bar{K}, K\bar{K}\pi, \pi\rho, \ldots\) + poles
  \(\rightarrow t_0 = 9M^2_\pi\)

- *Isovector* intermediate states: \(2\pi, 4\pi, \ldots\) + poles
  \(\rightarrow t_0 = 4M^2_\pi\)

- Note that some poles are *generated* from the appropriate continua
ISOVECTOR SPECTRAL FUNCTIONS

- exact $2\pi$ continuum is known from threshold $t_0 = 4M_\pi^2$ to $t \simeq 40M_\pi^2$

$$\text{Im } F_i^V(t) = \frac{q^3}{\sqrt{i}} |F_\pi(t)|^2 J_i(t)$$

$F_\pi(t)$ = pion vector form factor

$J_i \sim$ P-wave pion-nucleon partial waves in the t-channel

- Spectral functions inherit singularity on the second Riemann sheet in $\pi N \rightarrow \pi N$

$$t_c = 4M_\pi^2 - M_\pi^4/m^2 \simeq 3.98M_\pi^2$$

$\rightarrow$ strong shoulder $\rightarrow$ isovector radii

- This singularity can also be analyzed in CHPT

- Higher mass states represented by poles (with a finite width)

$\rightarrow$ not necessarily physical masses


For related work, see Dubnicka et al., J.Phys. G 29 (2003) 405
NEW DETERMINATION OF THE $2\pi$ CONTINUUM


- Pion FF from KLOE/CMD-2/SND

- Nucleon isovector spectral functions

- pronounced $\rho - \omega$ mixing


- pronounced $\rho$ peak
  - strong shoulder on the left wing

  ⇒ isovector radii
ISOSCALAR SPECTRAL FUNCTIONS

- $K\bar{K}$ continuum can be extracted from analytically cont. $KN$ scattering amplitudes
  - $\rightarrow$ analytic continuation must be stabilized
  - $\rightarrow$ generates most of the $\phi$ contribution


- Further strength in the $\phi$-region generated by correlated $\pi\rho$ exchange
  - $\rightarrow$ strong cancellations ($K\bar{K}, K^*K, \pi\rho$)
  - $\rightarrow$ takes away sizeable strength from the $\phi$


- Spectral functions exhibit anomalous threshold (analyzed in 2-loop CHPT)
  
  \[ t_c = M_\pi^2 \left( \sqrt{4 - M_\pi^2/m^2} + \sqrt{1 - M_\pi^2/m^2} \right)^2 \simeq 8.9 M_\pi^2 \]
  
  $\rightarrow$ effectively masked


- Higher mass states represented by poles (with a finite width)
CONSTRAINTS ON THE SPECTRAL FUNCTIONS

- Normalizations: electric charges, magnetic moments

- Superconvergence relations $\cong$ leading pQCD behaviour

\[
F_1(t) \sim 1/t^2, \quad F_2(t) \sim 1/t^3 \quad \text{(helicity – flip)}
\]

\[
\Rightarrow \int_{t_0}^{\infty} \text{Im} \ F_1(t) \, dt = 0, \quad \int_{t_0}^{\infty} \text{Im} \ F_2(t) \, dt = \int_{t_0}^{\infty} \text{Im} \ F_2(t) \ t \, dt = 0
\]

- Leading QCD logs or other $\alpha_S$ corrections can be included


\[
\Rightarrow \text{severely restricts the number of fit parameters}
\]
SUMMARY: SPECTRAL & FIT FUNCTIONS

• Representation of the pole contributions: vector mesons
  [NB: can be extended for finite width]

\[
\text{Im } F_i^V(t) = \sum_v \pi a_i^v \delta(t - M_v^2), \quad a_i^v = \frac{M_v^2}{f_V} g_{vNN} \Rightarrow F_i(t) = \sum_v \frac{a_i^v}{M_v^2 - t}
\]

• Isovector spectral functions:

\[
\text{Im } F_i^V(t) = \text{Im } F_i^{(2\pi)}(t) + \sum_{v=\rho',\rho'',...} a_i^v \delta(t - M_v^2), \quad (i = 1, 2)
\]

• Isoscalar spectral functions:

\[
\text{Im} F_i^S(t) = \pi a_i^\omega \delta(t - M_\omega^2) + \text{Im} F_i^{(K\bar{K})}(t) + \text{Im} F_i^{(\pi\rho)}(t) + \sum_{v=S',S'',...} a_i^v \delta(t - M_v^2)
\]

• Parameters: 2 for the \( \omega \), 3 (4) for each other \( V \)-mesons minus # of constraints

• Ill-posed problem \( \rightarrow \) extra constraint: minimal # of poles to describe the data
Results

GENERAL COMMENTS ON THE FITS

• large MC sampling for initial values, successive improvement by pole reduction, new MCs, . . .

• theoretical uncertainty (error bands) from $\chi^2_{\text{min}} + 1.04$ [1-$\sigma$ devs.]

$\rightarrow$ first time: dispersive analysis w/ error bars!

<table>
<thead>
<tr>
<th></th>
<th>this work</th>
<th>HM 04</th>
<th>recent determ.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$r^p_E$ [fm]</td>
<td>0.844 (0.840...0.852)</td>
<td>0.848</td>
<td>0.880(15) [1,2,3]</td>
</tr>
<tr>
<td>$r^p_M$ [fm]</td>
<td>0.854 (0.849...0.859)</td>
<td>0.857</td>
<td>0.855(35) [4]</td>
</tr>
<tr>
<td>$(r^n_E)^2$ [fm]</td>
<td>−0.117 (−0.11...−0.128)</td>
<td>−0.12</td>
<td>−0.115(4) [5]</td>
</tr>
<tr>
<td>$r^n_M$ [fm]</td>
<td>0.862 (0.854...0.871)</td>
<td>0.879</td>
<td>0.873(11) [6]</td>
</tr>
</tbody>
</table>

[2] Sick, private communication

★ Magnetic radii in good agreement with recent determinations
★ Proton electric radius comes out $\lesssim 0.855$ fm
SPACE-LIKE FORM FACTORS

- present best fit incl. time-like data
- 4 effective IS poles
- 4 effective IV poles
- weighted $\chi^2$/dof = 1.8
  error bands: $\chi_{\text{min}}^2 + 1.04$

Improved description
- JLab data described
- higher mass poles not at physical values

$G_D(Q^2) = \left(1 + \frac{Q^2}{0.71 \text{ GeV}^2}\right)^{-2}$

MMD 96, HMD 96, HM 04
SPACE-LIKE FORM FACTORS: NEW CLAS DATA

CLAS collaboration, to be published

→ apparent discrepancy to be resolved
TIME-LIKE FORM FACTORS

- fitting also time-like data more complicated
- experimental extraction ambiguous
  - E/M separation
  - $\bar{N}N$ final-state interactions?
    similar to $J/\psi \rightarrow \bar{p}p\gamma$ from BES
    similar to $B^+ \rightarrow \bar{p}pK^+$ from BaBar
  - subthreshold resonance ? (or FSI ?)
- many new proton data (radiative return)
  BES, CLEO, BaBaR

\[
\sigma(e^+e^- \leftrightarrow p\bar{p}, e^+e^- \rightarrow n\bar{n})
\]
• Only proton data participate in the fits

• All data within one sigma – first time consistent fit w/ space-like ffs

⇒ Need more data on time-like $G_M^n$
• FW find a very long-ranged contribution of the pion could, \( r \simeq 2 \text{ fm} \)
  Friedrich, Walcher, EPJ A 17 (2003) 607

• longest range component can be extracted from the isovector spectral function
  → separation of the \( \rho \)-contribution
  → three methods applied to do this
  → theoretical band

\[
\rho_i^V(r) = \frac{1}{4\pi^2} \int_{4M_\pi^2}^{40M_\pi^2} dt \, \text{Im} \, G_i^V(t) \frac{e^{-r\sqrt{t}}}{r} \quad (i = E, M)
\]

• much smaller pion cloud contribution for \( r \geq 1 \text{ fm} \) compared to FW
• results independent of the contributions from \( t > 40M_\pi^2 \)
$G_{E}^{n}(Q^{2})$ w/ a BUMP-DIP STRUCTURE

- can one generate a bump-dip structure in the dispersive approach?

⇒ yes, but need low-mass poles: $M_{S} = 358$ MeV & $M_{V} = 558$ MeV

what shall these be? – not consistent w/ spec ftcs!
SUMMARY & OUTLOOK

• New dispersive analysis of the nucleon em form factors

• Improved spectral functions \( \Rightarrow \) many results
  – better fits w/ inclusion of time-like form factors
  – theoretical/systematic uncertainty \( \rightarrow \) bands

• Still to be done
  – including pQCD corrections at large-\( t \) beyond SCR
  – two-photon effects? \( \rightarrow \) fit to X sections
  – consequences for the strangeness vector form factors
  – and much more . . .